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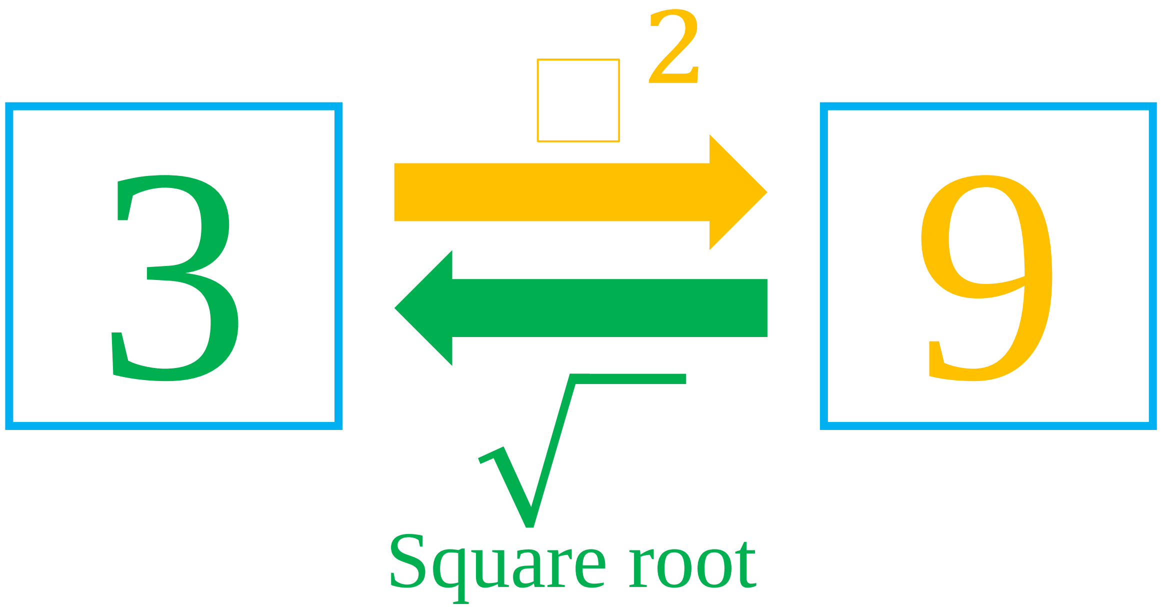
Real numbers

Part 5 (Square root)

Introduction

Recall that a square of a number is the result of multiplying this number by itself.

Example: $3^2 = 3 \times 3 = 9$



As a result, square root of a number a is a value that is multiplied by itself to obtain the original number a.

It is denoted $\sqrt{a}$ that is read radical of a

Introduction

Determine the square root of each number in each case.

The number a	$\sqrt{a}$
4	$\sqrt{4} = 2$
9	$\sqrt{9} = 3$
16	$\sqrt{16} = 4$
81	$\sqrt{81} = 9$
144	$\sqrt{144} = 12$
2500	$\sqrt{2500} = 50$

Since $2^2 = 4$

Since $3^2 = 9$

Since $4^2 = 16$

Since $9^2 = 9$

Since $12^2 = 144$

Since $50^2 = 2500$

IMPORTANT

- If a number a is negative, then $\sqrt{a}$ doesn't exist.

Example:

$\sqrt{-4}$ doesn't exist since there is no number that is multiplied by itself gives -4: $2 \times 2 = 4 \neq -4$ and $(-2) \times (-2) = 4 \neq -4$

- $\sqrt{a}$ is a positive number
- $\sqrt{a^2} = \begin{cases} a & if\ a \geq 0 \\ opp(a) = -a & if\ a \leq 0 \end{cases}$

Example:

$\sqrt{3^2} = 3$; $\sqrt{(-16)^2} = -(-16) = 16$

Fundamental rules

$a > 0$ & $b > 0$

❖ $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

Example:

$\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$

$\sqrt{8} \times \sqrt{2} = \sqrt{8 \times 2} = \sqrt{16} = 4$

❖ $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Example:

$\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}} = \frac{2}{5}$; $\frac{\sqrt{3}}{\sqrt{27}} = \sqrt{\frac{3}{27}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

IMPORTANT

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- $\sqrt{a}+\sqrt{b} \neq \sqrt{a+b}$
- $\sqrt{a}-\sqrt{b} \neq \sqrt{a-b}$

Example:

$\sqrt{16}+\sqrt{9}=4+3=7$ but $\sqrt{16+9}=\sqrt{25}=5 \neq 7$

$\sqrt{25}-\sqrt{9}=5-3=2$ but $\sqrt{25-9}=\sqrt{16}=4 \neq 2$

Fundamental rules

$a > 0\,; b > 0\,; c > 0\ \&\ d > 0$

❖ $\sqrt{a} \times \sqrt{a} = (\sqrt{a})^2 = a$

Example:

$(\sqrt{2})^2 = 2 \quad ; \quad (\sqrt{5})^2 = 5$

❖ $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$

Example:

$2\sqrt{3} \times 5\sqrt{2} = 2 \times 5\sqrt{3 \times 2} = 10\sqrt{6}$

$-3\sqrt{3} \times \sqrt{3} = -3 \times 3 = -9$

Fundamental rules

$a > 0\,; b > 0\,; c > 0\ \&\ d > 0$

- ❖ $a\sqrt{b} + c\sqrt{b} = (a + c)\sqrt{b}$
- ❖ $a\sqrt{b} - c\sqrt{b} = (a - c)\sqrt{b}$

Example:

$2\sqrt{3} + 3\sqrt{3} = (2 + 3)\sqrt{3} = 5\sqrt{3}$

$-5\sqrt{2} + 3\sqrt{2} + \sqrt{2} - 5\sqrt{2} = (-5 + 3 + 1 - 5)\sqrt{2}$

Simplify expressions containing radicals

Example 1:

Simplify $A = \sqrt{2^4}$

To simplify, divide the exponent by 2

Then $A = 2^2$

Simplify $A = \sqrt{2^6}$

$A = 2^3$

Simplify $A = \sqrt{2^8 \times 3^{16} \times 25}$

$A = 2^4 \times 3^8 \times 5$

Simplify $A = \sqrt{(-7)^2 \times 3^4}$

$A = 7 \times 3^2$

As a result if the exponent is even, just divide it by 2 to get the answer.

But pay attention, the answer will be positive.

Example: $\sqrt{(-7)^2} = 7$

Simplify expressions containing radicals

Example 2:

Simplify $A = \sqrt{2^5}$

To simplify, decompose the exponent into two numbers 1 and the even number just before the exponent: 5 = 4+1

Then $A = \sqrt{2^4 \times 2} = 2^2\sqrt{2}$

Simplify $A = \sqrt{2^{101}}$

$A = \sqrt{2^{100} \times 2} = 2^{50}\sqrt{2}$

Simplify $A = \sqrt{2^8 \times 3^{25} \times 49}$

$A = 2^4 \times 7 \times \sqrt{3^{24} \times 3} = 2^4 \times 7 \times 3^{12}\sqrt{3}$

As a result if the exponent is odd, decompose it into "1 + even number". Example: 3=2+1 ; 5=4+1 ; 9=8+1 …

Simplify expressions containing radicals

Example 3:

Simplify $A = \sqrt{72}$

To simplify, decompose using the prime number decomposition.

72	2	2^2
36	2	
18	2	
9	3	3^2
3	3	
1		

$$A = \sqrt{2^2 \times 2 \times 3^2} = 2 \times 3\sqrt{2} = 6\sqrt{2}$$

Simplify expressions containing radicals

Example 3:

Simplify $A = \sqrt{84}$

To simplify, decompose using the prime number decomposition.

84	2
42	2
21	3
7	7
1	

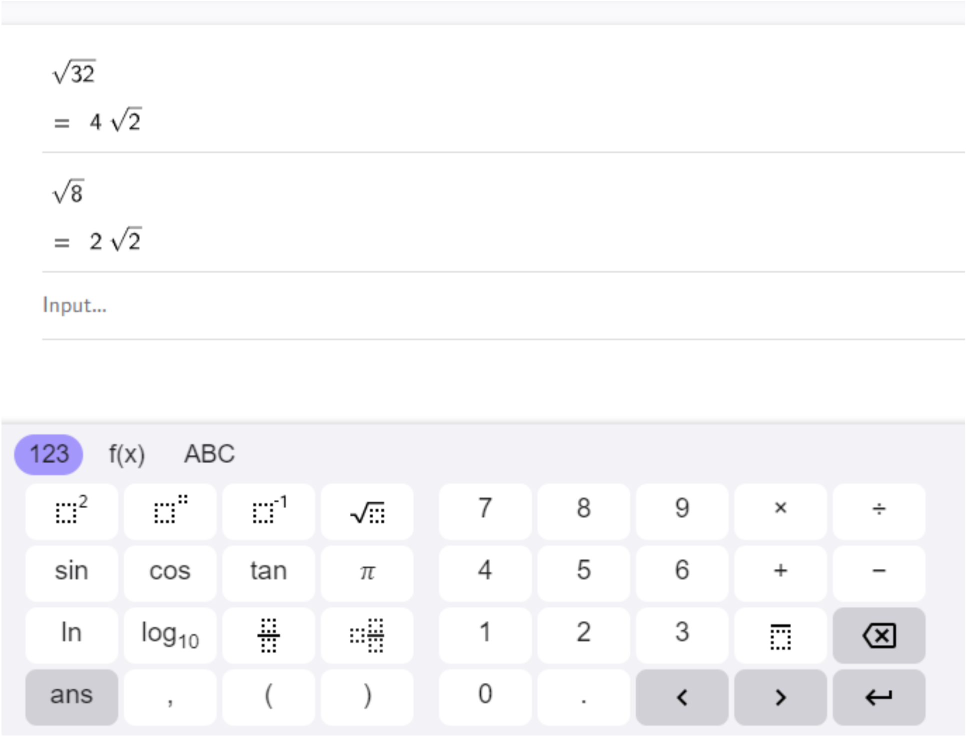
2^2

$$A = \sqrt{2^2 \times 3 \times 7} = 2\sqrt{3 \times 7} = 2\sqrt{21}$$

Simplify expressions containing radicals

IMPORTANT

In the last examples, we've learned to simplify expressions containing radicals using the best method: "prime factorization" But you can use simply the calculator to simplify $\sqrt{a}$.



Example:

Simplify $A = \sqrt{32} + \sqrt{8}$

Using calculator: $A = 4\sqrt{2} + 2\sqrt{2} = (4 + 2)\sqrt{2} = 6\sqrt{2}$.

Application

Write $A = 2\sqrt{3} + 6\sqrt{12} - 2\sqrt{27} + 3\sqrt{48}$ in form of $a\sqrt{b}$ where a is an integer and b is a natural number.

$$A = 2\sqrt{3} + 6\sqrt{12} - 2\sqrt{27} + 3\sqrt{48}$$
$$= 2\sqrt{3} + 6 \times 2\sqrt{3} - 2 \times 3\sqrt{3} + 3 \times 4\sqrt{3}$$
$$= 2\sqrt{3} + 12\sqrt{3} - 6\sqrt{3} + 12\sqrt{3}$$
$$= (2 + 12 - 6 + 12)\sqrt{3}$$
$$= 20\sqrt{3}$$

Application

Expand and reduce the following expression.

$$A = \left(2\sqrt{3} + 2\right)\left(2\sqrt{3} - 2\right)$$

$$A = \left(2\sqrt{3} + 2\right)\left(2\sqrt{3} - 2\right)$$ using remarkable identity $(a - b)(a + b) = a^2 - b^2$

$$= \left(2\sqrt{3}\right)^2 - 2^2$$

$$= 12 - 4$$

$$= 8$$